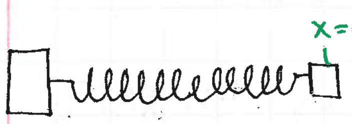
CH. 15 May 6, 2019

‖CHAPTER 15: OSCILLATIONS‖

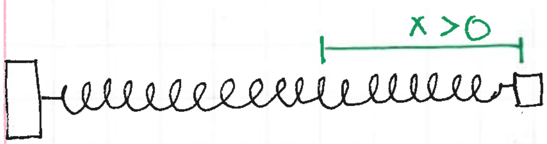
Any system where the net force has the form $F_{net} = -kx$ undergoes a periodic motion called Simple harmonic motion (SHM)

Periodic motion in which the position, velocity, & acceleration can be described by either a sin or cos function.

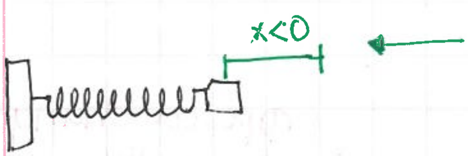
Simple System → mass connected to a spring of spring constant k on a frictionless surface



(Equilibrium position → Spring is not stretched or compressed)



x > 0 spring is stretched



x < 0 spring is compressed

amplitude (A) → Greatest distance from the equilibrium position (m)

Cycle → One complete oscillation or one complete back & forth motion

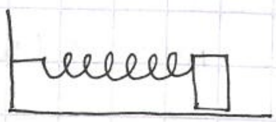
CH.15 — May 6, 2019

Period (T) → Time for one complete cycle (s)

frequency (f) → number of oscillations per second ($Hz = cycle/s = s^{-1}$)

$\boxed{T = 1/f}$ $\boxed{f = \frac{1}{T}}$

Equation of motion for SHM:



frictionless

$\sum F_x = ma_x$

$-kx = ma_x$

$m\frac{d^2x}{dt^2} = -kx$

$\frac{d^2x}{dt} = -\frac{k}{m}x$

$\boxed{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$

↳ differential equation for SHM

A function that when you take a derivatives gives the original function times some constant.

↳ Sin or cos function

general solution:

$\boxed{x(t) = A\cos(\omega t + \phi_0)}$

A → amplitude in meters

ω → angular frequency in $\frac{rad}{s}$

t → Elapsed time in seconds

ϕ_0 → phase angle or phase constant.

↓ Describes the initial conditions of the system (what are x_0 and v_{0x})

CH 15 May 6, 2019

$$\frac{d^2x}{dt^2} = \frac{-k}{m}x$$

$$x(t) = A\cos(\omega t + \phi_0)$$

$$\frac{dx}{dt} = -A\omega\sin(\omega t + \phi_0) \longrightarrow v(t) = -A\omega\sin(\omega t + \phi_0)$$

$$\boxed{(v_t) = -v_{max}\sin(\omega t + \phi_0)}$$

$$\boxed{v_{max} = \omega A}$$

$$\frac{d^2x}{d^2t} = -A\omega^2\cos(\omega t + \phi_0) \longrightarrow a(t) = -A\omega^2\cos(\omega t + \phi_0)$$

$$\boxed{a(t) = -a_{max}\cos(\omega t + \phi_0)}$$

$$\boxed{a_{max} = \omega^2 A}$$

$$\frac{d^2x}{dt^2} = -kx$$

$$-A\omega^2\cos(\omega t + \phi_0) = \frac{-k}{m}A\cos(\omega t + \phi_0)$$

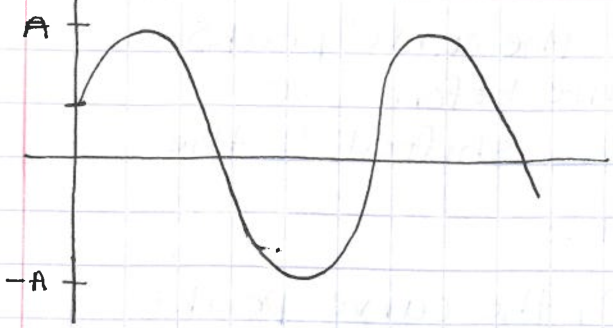
Will be true if $\omega^2 = \frac{k}{m}$ or

$\omega = \sqrt{\frac{k}{m}}$

$$\boxed{\begin{array}{c} x(t) = A\cos(\omega t + \phi_0) \\ \omega = \sqrt{\frac{k}{m}} \end{array}}$$

CH.15 May 6, 2019

Conceptual 15.4A



$x(t) = A\cos(\omega t + \phi_0)$

at $t=0$, $x = \frac{1}{2}A$

$\frac{1}{2}A = A\cos\phi_0$

$\cos\phi_0 = \frac{1}{2}$

$\phi_0 = \cos^{-1}\left(\frac{1}{2}\right)$

$\phi = \pm\pi/3$

$x(t=0) = A\cos(\omega(0) + \phi_0)$

$x(t=0s) = A\cos\phi_0$

$\cos\phi = \cos(-\phi)$

phase: $\phi = \omega t - \phi_0 \rightarrow$

$0 \le \phi \le \pi/2$ — $x>0$, $v<0$

$\pi/2 < \phi < \pi$ — $x<0$, $v<0$

$\pi < \phi < 3\pi/2$ — $x<0$, $v>0$

$\frac{3\pi}{2} < \phi < 2\pi$ — $x>0$, $v>0$

* The phase angle should be equal to $-\pi/3$ b/c $v>0$ at $t=0$

$\phi_0 = -\pi/3$

Phase angle

$x(t) = A\cos(\omega t + \phi_0)$

when is object at $x = A$?

$A = A\cos(\omega t + \phi_0)$

$\cos(\omega t + \phi_0) = 1$

$\omega t + \phi = 0$

$t = \frac{-\phi}{\omega}$

CH.15, RECAP May 6, 2019

$\phi > 0$ $t < 0$ * If $\phi > 0$ the curve peaks at a time before $t = 0$ (curve is shifted to the left)

$\phi < 0$ $t > 0$ * If $\phi < 0$, the curve peaks at a time after $t = 0$ (curve is shifted to the right)

RECAP May 7, 2019

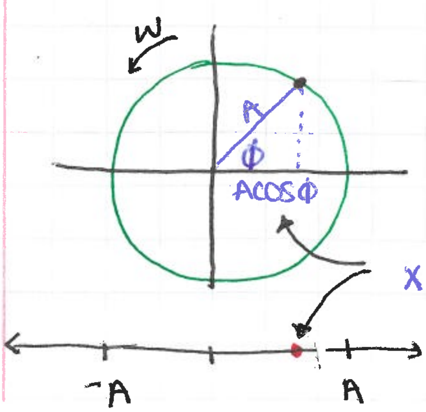
Simple Harmonic → motion (SHM) periodic motion that a system of the form $\underline{\Sigma F = -kx}$ undergoes

$$\left(\frac{d^2x}{dt^2} = -\frac{k}{m} x \right)$$

↳ Differential equation for SHM

General Solution → $\boxed{x(t) = A\cos(\omega t + \phi)}$

* SHM is the projection onto the x-axis of uniform circular motion.



x - position of particle describes the position of the ball's shadow.

CH15 May 7, 2019

RECAP

$$\boxed{x(t) = A\cos(\omega t + \phi_0)}$$

$$v(t) = -A\omega\sin(\omega t + \phi_0)$$
$$= -v_{max}\sin(\omega t + \phi_0)$$

$$\boxed{v_{max} = A\omega}$$

$$a(t) = -A\omega^2\cos(\omega t + \phi_0)$$
$$= -a_{max}\cos(\omega t + \phi_0)$$

$$\boxed{a_{max} = A\omega^2}$$

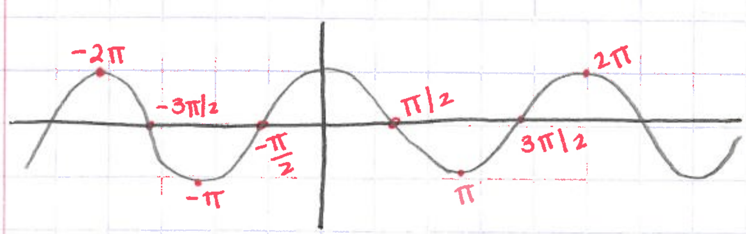
$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

$\omega \rightarrow$ angular frequency (rad/s)
$t \rightarrow$ time in seconds
$\phi_0 \rightarrow$ phase constant (rad)
$\phi \rightarrow \omega t + \phi_0$ is called the phase (angle in rad)

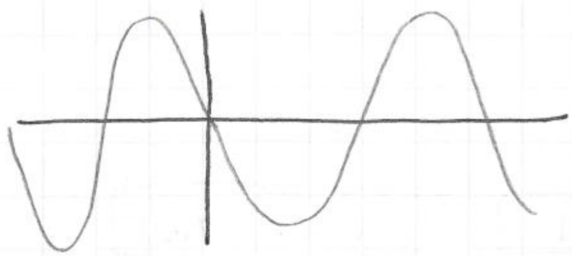
Phase ϕ

I	$x > 0$	$v < 0$
II	$x < 0$	$v < 0$
III	$x < 0$	$v > 0$
IV	$x > 0$	$v > 0$

* If $\phi_0 > 0$, curve is shifted to the left
If $\phi_0 < 0$, curve is shifted to the right.



CH 15. May 7, 2019



What is ϕ_0?

$\phi_0 = +\pi/2$

$= -3\pi/2$

Problem 15.6

(a) 10cm

(b) 0.25 Hz = $\frac{1}{4}$ Hz

(c) at $t=0$, $x_0 = -5cm$

$x(t) = A\cos(\omega t + \phi_0)$

$-5cm = 10cm \cos(\phi_0)$

$\cos\phi_0 = \frac{-5cm}{10cm} \rightarrow \phi_0 = \cos^{-1}(-\frac{1}{2})$

$\phi_0 = 120° = \frac{2\pi}{3}$ rad

* $\cos\phi = \cos(-\phi)$

$\Rightarrow \phi_0 = \pm 2\pi/3$

$\boxed{\phi_0 = -\frac{2\pi}{3}}$

$v(t=0) = -A\omega\sin\phi_0$

CH 15 May 7, 2019

$x(t) = A\cos(\omega t + \phi_0)$

↳ Motion repeats itself every period

↳ Cosine repeats itself every 2π

$\cos(\omega(t+T) + \phi_0) = \cos(\omega t + 2\pi + \phi_0)$

$\omega(t+T) = \omega t + 2\pi$

$\omega t + \omega T = \omega t + 2\pi$

$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$

$\boxed{\omega = \frac{2\pi}{T}}$ $\boxed{\omega = 2\pi f}$ $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$ $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

* There are independent * of amplitude

Problem 15.11

$A = 4.0\,cm$

$f = 4.0\,Hz$

$\omega = 2\pi f = 8\pi\,rad/s$

$x_0 = x(t = 0s) = 0\,cm$

$v_0 > 0$

$0\,cm = (4\,cm)\cos\phi_0$

$\cos\phi_0 = 0$

$\phi_0 = \pm\pi/2$

since $v > 0$ $\phi_0 < 0$

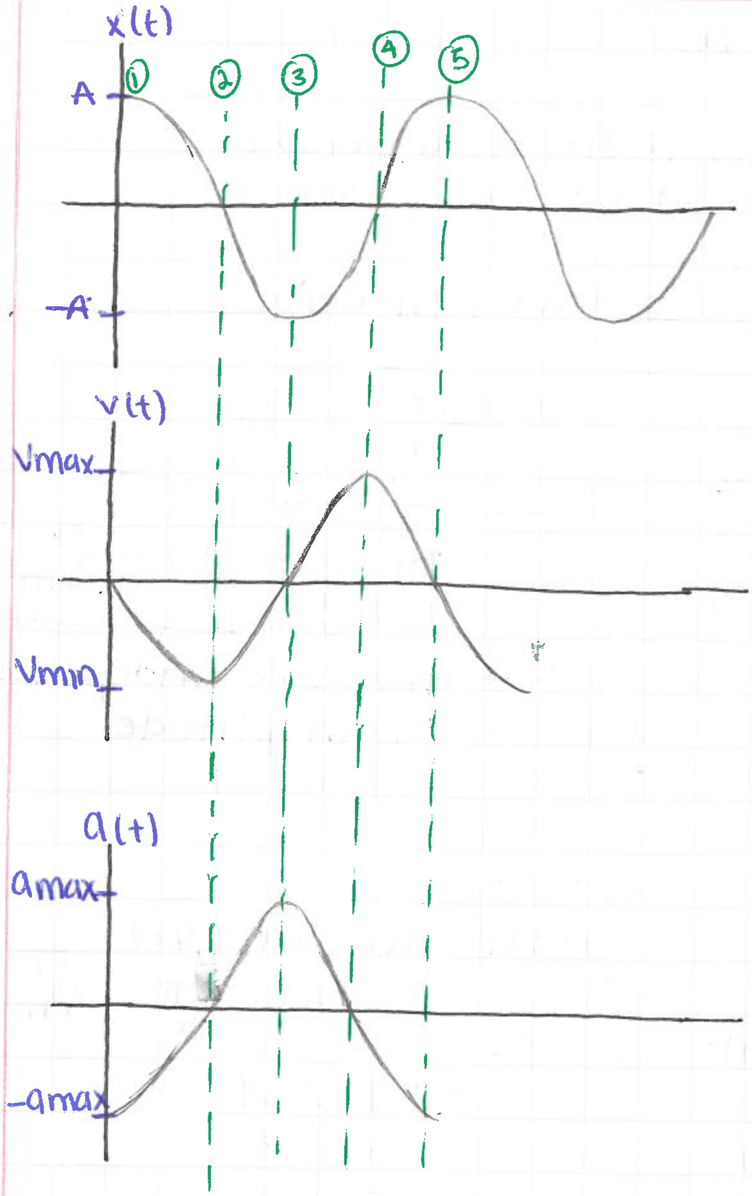
$x(t) = A\cos(\omega t + \phi_0)$

$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$x_0 = A\cos\phi_0$

$\boxed{x(t) = (4.0\,cm)\cos\left[\left(8\pi\frac{rad}{s}\right)t - \frac{\pi}{2}\right]}$

CH. 15 May 7, 2019

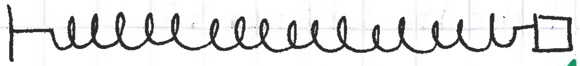


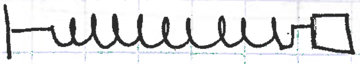
$x(t) = \underline{A}\cos(\omega t + \phi_0)$

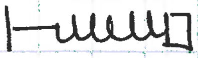
$v(t) = \underline{-A\omega}\sin(\omega t + \phi_0)$

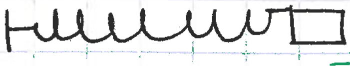
$a(t) = \underline{-A\omega^2}\cos(\omega t + \phi_0)$

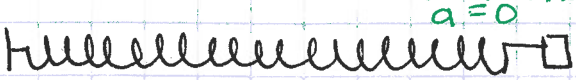
CH. 15 May 7, 2019

1  $V=0$
$a=-a_{max}$

2  $V=-V_{max}$
$a=0$

3  $V=0$
$a=+a_{max}$

4 
$V=+V_{max}$
$a=0$

5 

$a(t) = -\omega^2 x(t)$

t	x	v	a	
0	A	0	$-a_{max}$	1
T/4	0	$-V_{max}$	0	2
T/2	−A	0	a_{max}	3
3T/4	0	$+V_{max}$	0	4
T	A	0	$-a_{max}$	5

ENERGY IN SHM

If there is No friction or air resistance, then the only forms of energy is horizontal SHM are kinetic energy & Elastic potential Energy

$$\boxed{E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2}$$

- Is the energy ever all kinetic? Potential? Where and what is E?

  - at $x=0$, E is all kinetic $\boxed{E=\frac{1}{2}mv^2_{max}}$

  - at $x=\pm A$, $v=0$ & Energy is all potential $\boxed{E=\frac{1}{2}kA^2}$

$$E=\frac{1}{2}mv^2+\frac{1}{2}kx^2$$
$$=\frac{1}{2}mv^2_{max}$$
$$=\frac{1}{2}kA^2$$

Problem 15.18

$m=1.0kg$; $k=16\,N/m$; at $x=0\,m$ $v=4.40\,m/s$
$A=?$

- a) $\frac{1}{2}mv^2+\frac{1}{2}\cancel{kx^2}^{\,0}=\overset{E}{\frac{1}{2}kA^2} \rightarrow \frac{1}{2}mv^2=\frac{1}{2}kA^2$

$$A=\sqrt{\frac{m}{k}}\,v \qquad A=\sqrt{\frac{(1.0kg)}{(16N/m)}}\,(4.0m/s) \rightarrow A=10cm$$
$$=\underline{0.10cm}$$
$$\frac{1}{2}A=\underline{0.05m}$$

- b) $v=?$ $x=\frac{1}{2}A$

$$\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$$
$$\frac{1}{2}mv^2=\frac{1}{2}k(A^2-x^2) \rightarrow v^2=\frac{k}{m}(A^2-x^2)$$
$$v=\sqrt{\frac{k}{m}(A^2-x^2)} \longrightarrow v=\sqrt{\frac{16N/m}{1.0kg}[(0.10m)^2-(0.050m)^2]}$$
$$v=0.340\,m/s \rightarrow \boxed{35\,cm/s}$$

CH.15 + Recap — May 9 2019

Recap

$$\boxed{v_{max} = A\omega}$$
$$\boxed{a_{max} = A\omega^2}$$

$$x(t) = A\cos(\omega t + \phi_0)$$
$$v(t) = -A\omega\sin(\omega t + \phi_0) = -v_{max}\sin(\omega t + \phi_0)$$
$$a(t) = -A\omega^2\cos(\omega t + \phi_0) = -a_{max}\cos(\omega t + \phi_0)$$

True for any SHM-system

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}}$$

Only true for Spring-mass system

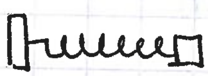
$\phi_0 > 0$ Curve shifted left
$\phi_0 < 0$ Curve shifted Right

Note: $\boxed{a(t) = -\omega^2 x(t)}$

Hallmark of SHM

* If a system has the form $a(t) = -(\text{constant})\,x(t)$, it will undergo SHM with angular frequency $\omega = \sqrt{\text{constant}}$

Ex



$$F = ma = -kx$$
$$a = -\frac{k}{m}x$$
$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

ENERGY IN S.H.M.

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$
$$= \frac{1}{2}mv_{max}^2 \quad \text{Equilibrium}$$
$$= \frac{1}{2}kA^2 \quad \text{End points}$$

CH 15

May 9, 19

Problem 15.16

$m = 0.200kg$
$f = 2.0Hz$
at $t = 0s$, $x_0 = 5.0cm$
$V_{0x} = -30\ cm/s$

a) The Period b) angular frequency, c) the amplitude
d) The phase constant e) The maximum speed
f) the maximum acceleration g) The total energy
h) the position at $t = 0.40s$

a) $T = 1/f = 1/2.0Hz = \underline{0.50s}$

b) $\omega = 2\pi f = 2\pi(2.0Hz) = \underline{4\pi rad/s}$

c) $E = \frac{1}{2}mv^2 + \frac{1}{2}kx = \frac{1}{2}kA^2$

at $x = \pm A$, $v = 0 \rightarrow E = \frac{1}{2}kA^2$
at $x = 0$, $v = v_{max} \rightarrow E = \frac{1}{2}mV_{max}^2$

$\omega = \sqrt{\frac{k}{m}}$ $\omega^2 = \frac{k}{m} \rightarrow k = m\omega^2$

$k = (0.200kg)(4\pi rad/s)^2 = 31.6N/m$

$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$A^2 = \frac{m}{k}V^2 + x^2 \longrightarrow A = \sqrt{\frac{m}{k}V^2 + x^2} \rightarrow A = \underline{5.54cm}$

d) $x(t) = A\cos(\omega t + \phi_0)$ @ $t=0 \longrightarrow x_0 = A\cos\phi_0$
$V(t) = -A\omega\sin(\omega t + \phi_0)$ at $t=0 \longrightarrow V_{0x} = -A\omega\sin\phi_0$

$\dfrac{V_{0x} = -A\omega\sin\phi_0}{x_0 = A\cos\phi_0} \longrightarrow \dfrac{V_{0x}}{x_0} = -\omega\tan\phi_0$

$\phi_0 = \tan^{-1}\left(\frac{V_{0x}}{\omega x_0}\right)$

OR d) at $t = 0s$, $x_0 = 5.00m$

$A = 5.50 cm$

$x_0 = A\cos\phi_0$

$\cos\phi_0 = \frac{x_0}{A} = \frac{5.0cm}{5.54cm} \rightarrow \phi_0 = \cos^{-1}\left(\frac{5.0\ cm}{5.54cm}\right) \rightarrow 0.45rad$

$\cos\theta = \cos(-\theta)$

$\phi_0 = \pm 0.45 rad$

$\phi_0 = 0.45 rad$

e) $V_{max} = A\omega = 70 cm/s$

f) $a_{max} = A\omega^2 = 8.8 m/s^2$

g) $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \rightarrow$ x & v at t=0

$E = \frac{1}{2}kA^2$ $E = 0.049J$

$E = \frac{1}{2}mV_{max}^2$

h) $x(t) = A\cos(\omega t + \phi_0)$

$x(t) = (5.54 cm)\cos[(4\pi rad/s)t + 0.45 rad]$

$x(t=0.40s) = (5.54cm)\cos[(4\pi rad/s)(0.40s) + 0.45rad]$

$= 3.8cm$

1Q. Object A is attached to ideal spring A and is moving in simple harmonic motion. Object B is attached to ideal spring B and is moving in SHM. The period and the amplitude of object B are both 2x the corresponding values for object A. How do the max. speeds of the two compare? They are the same.

$A_A = 2A_B$ $V_{max} = A\omega$

$T_A = 2T_B$

Small Angle Approximation

$\boxed{\text{If } \theta \ll 1 \text{, then } \sin\theta \simeq \theta \quad (\theta \text{ is in radians})}$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - + \ldots$$

If $\theta \ll 1$, then we can ignore the $\theta^3, \theta^5, \ldots$ terms because they are vanishingly small.

$\theta = 20°$

$\theta = 20° \left(\frac{2\pi \text{rad}}{360}\right) = 0.3491 \text{rad}$

$\sin\theta \simeq \theta$

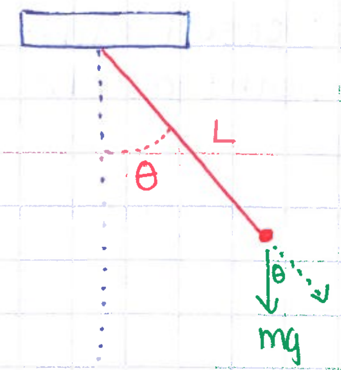
approximation → $\sin(0.3491 \text{ rad}) \simeq 0.3491$

actual $\sin(0.3491) = 0.3420$ 2% difference

Pendulums

↳ A pendulum swings at small amplitude oscillations exhibits SHM

Simple Pendulum → A particle of mass m suspended from one end of a massless, unstretchable string of length L.



derive different than book

$\sum \tau = I\alpha$, $\tau = rF\sin\theta$

$-L(mg)\sin\theta = I\alpha$

for small angles $\rightarrow \sin\theta \simeq \theta$

$-Lmg\theta = I\alpha$

$\alpha = -\left(\frac{Lmg}{I}\right)\theta$ → angular equivalent to $a = -\omega^2 x$

for a simple pendulum swinging at small amplitudes:

$\omega = 2\pi f = \sqrt{\frac{g}{L}}$

$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$

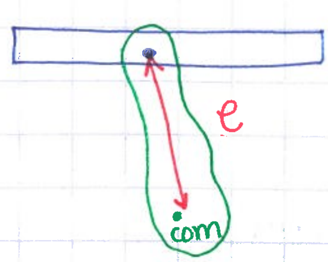
$\boxed{T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}}$

$\omega^2 = \frac{Lmg}{I} \rightarrow \omega = \sqrt{\frac{Lmg}{I}}$

Note: $I = mL^2$

$\omega = \sqrt{\frac{\cancel{L}\cancel{m}g}{\cancel{m}L^{\cancel{2}}}} \rightarrow \omega = \sqrt{\frac{g}{L}}$

Physical Pendulum



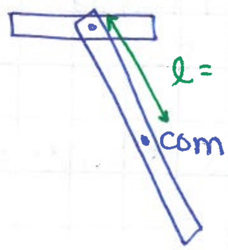
physical pendulum

$\boxed{\omega = \sqrt{\frac{\ell mg}{I}}}$

→ only real difference in analyzing a real pendulum (physical pendulum) is that the gravitational force acts at the com a distance ℓ from the pivot point (and $I \neq mL^2$)

May 9, 2019

Ex What is T for a meter stick oscillating back & forth with pivot point at one end?



$L = 1.0m$

$$\omega = \sqrt{\frac{\ell m g}{I}} \qquad \ell = L/2 \qquad I = \frac{1}{3} m L^2$$

$$\omega = \sqrt{\frac{(L/2)\, mg}{1/3\, mL^2}} = \sqrt{\frac{3g}{2L}} \qquad T = \frac{2\pi}{\omega}$$

$$T = 2\pi \sqrt{\frac{2L}{3g}} = 2\pi \sqrt{\frac{2(1.0m)}{3(9.80m/s^2)}} = 1.64s$$

Problem 15.26

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Answer/solution to 15.7a are wrong

$A = \phi_0 = -\pi/3$

not $\theta_0 = -2\pi/3$

May 2019

<u>1Q</u> A simple pendulum of length L and mass M has frequency F. To increase its frequency to 2f:
decrease its length to L/4